SOLUTION

a. The function could be quadratic because it has a vertex.

b. The function is increasing for $x < 3$ and decreasing for $x > 3$, and it is concave down.

c. The function could model anything that rises to a maximum and then falls back down again, such as the height of a ball as a function of time or the grade you could earn on a final exam as a function of how long you study for it. (Cramming too long might lower your score because of your being sleepy from staying up late!)

d. $y = ax^2 + bx + c$ Write the general equation.

$$\begin{cases} 76 = a + b + c \\ 89 = 4a + 2b + c \\ 94 = 9a + 3b + c \end{cases}$$ Substitute the given x- and y-values.

$$\begin{bmatrix} 1 & 1 & 1 \\ 4 & 2 & 1 \\ 9 & 3 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 76 \\ 89 \\ 94 \end{bmatrix} = \begin{bmatrix} -4 \\ 25 \\ 55 \end{bmatrix}$$ Solve by matrices.

$\therefore y = -4x^2 + 25x + 55$ Write the equation.

e. Plotting the graph confirms that the equation is correct. Note that the value of a is negative, which corresponds to the fact that the graph is concave down. ◀

EXAMPLE 3 ➤ For the function graphed in Figure 2-2h,

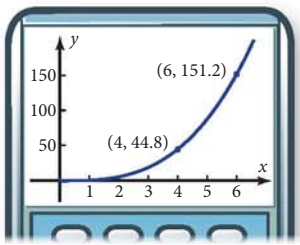

Figure 2-2h

a. Identify the kind of function it could be.

b. On what interval or intervals is the function increasing or decreasing? Which way is the graph concave, up or down?

c. Describe something in the real world that a function with this shape graph could model.

d. Find the particular equation of the function you identified in part a, given that points (4, 44.8) and (6, 151.2) are on the graph.

e. Confirm by plotting that your equation gives the graph in Figure 2-2h.

SOLUTION

a. The function could be a power function or an exponential function, but a power function is chosen because the graph appears to contain the origin. Exponential functions don't contain the origin unless they are translated in the y-direction.

b. The function is increasing and concave up over the entire domain shown.

c. The function could model anything that starts at zero and increases at an increasing rate, such as the power generated by a windmill as a function of wind speed, when the driver applies the brakes, or the volumes of geometrically similar objects as a function of their lengths.